Characterization and construction of singular distribution functions for random base-q expansions whose digits generate a stationary process

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Let F be the cumulative distribution function (CDF) of the base-q expansion $\sum_{n=1}^{\infty} X_n q^{-n}$, where $q \geq 2$ is an integer and $\{X_n\}_{n\geq 1}$ is a stochastic process with state space $\{0, \ldots, q-1\}$. We show that stationarity of $\{X_n\}_{n\geq 1}$ is equivalent to a certain functional equation obeyed by F, which enables us to give a complete characterization of the structure of F. In particular, we prove that the absolutely continuous component of F can only be the uniform distribution on the unit interval while its discrete component can only be a countable convex combination of certain explicitly computable CDFs for probability distributions with finite support. Moreover, we show that for a large class of stationary stochastic processes, their corresponding F is singular (that is, $F' = 0$ almost everywhere) and continuous; and often also strictly increasing on $[0, 1]$. We also consider geometric constructions and 'relatively closed form expressions' of F. Finally, we study special cases of models, including stationary Markov chains of any order, stationary renewal point processes, and mixtures of such models, where expressions and plots of F will be exemplified.

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