Ordered Random Walks

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June 6, 2012

We construct the conditional version of k independent and identically distributed random walks on the real line given that they stay in strict order at all times. This is a generalisation of so-called non-colliding or non-intersecting random walks, the discrete variant of Dyson's Brownian motions, which were considered before only for nearest- neighbor walks on the lattice. Our only assumptions are moment conditions on the steps and the validity of the local central limit theorem. The conditional process is constructed as a Doob h-transform with some positive regular function V that is strongly related with the Vandermonde determinant and reduces to that function for simple random walk. Furthermore, we prove an invariance principle, i.e., a functional limit theorem towards Dyson's Brownian motions, the continuous analogue. We also briefly describe some variants of similar conditionings of random walks and Brownian motions. (joint works with Peter Eichelsbacher and Patrick Schmid).