Seminar on stochastic processes

Random spatial networks

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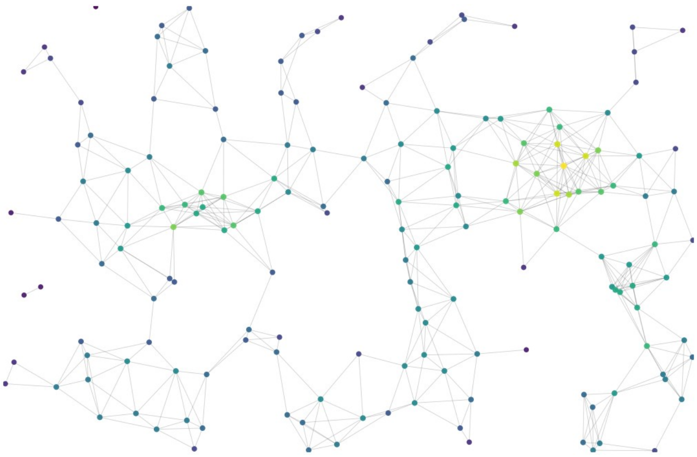
Christian Hirsch (Aarhus University)



A random spatial network is a graph whose vertices carry locations in space and whose edges are influenced by these locations. Geometry may enter through a hard distance threshold, through a distance-dependent connection probability, through nearest-neighbour rules, or through a random spatial environment. The resulting models combine random graph theory with stochastic geometry and percolation theory. They provide a natural framework for systems in which interactions are local or distance dependent, such as wireless and device-to-device networks, neuronal and biological networks, transport systems, porous materials, and spatially embedded social or collaboration networks.

As in classical random graph theory, we are primarily interested in large networks and in asymptotic questions. However, the presence of geometry changes the probabilistic structure substantially. In the Erdős-Rényi random graph different edges are independent, while in geometric graphs nearby edges often depend on common point locations. Triangles and other small subgraphs arise naturally from spatial proximity, and the local structure is therefore typically much more clustered. At the same time, global questions such as the appearance of a giant component, full connectivity, graph distances, or the existence of an infinite component are strongly constrained by the underlying geometry.

The seminar will begin with a small number of classical random graph models that serve as benchmarks for the spatial theory. We first use branching-process ideas to understand component exploration in the Erdős-Rényi graph. We then introduce rank-one inhomogeneous models, including the Norros-Reittu model, where random vertex weights create heterogeneous and possibly power-law degrees. Preferential attachment provides a complementary dynamic mechanism for producing heavy-tailed degree distributions. These models give us reference points for asking later what changes when geometry is added to an otherwise similar network mechanism.

The main part of the seminar concerns spatial networks. We first introduce Poisson point processes and the random geometric graph, where two random points are connected when they are sufficiently close. Continuum percolation studies the transition between a regime with only finite components and one with an unbounded component, while the random connection model replaces the hard distance threshold by a distance-dependent connection probability. We also develop stabilization methods for geometric graph statistics. These methods turn many global quantities into sums of local contributions and lead to laws of large numbers and central limit theorems for a broad class of random spatial graphs.

The next group of talks combines geometry with the heterogeneity mechanisms encountered at the beginning of the seminar. Weight-dependent random connection models attach random weights to Poisson points and let the edge probability depend jointly on the weights and spatial distance. Geometric inhomogeneous random graphs and hyperbolic random graphs provide closely related finite-network models that combine power-law degrees, short graph distances and strong clustering. Comparing these models with Norros-Reittu graphs makes it possible to isolate the effect of geometry from the effect of degree heterogeneity.

Finally, we consider spatial preferential attachment, where new vertices arrive over time and attachment depends simultaneously on spatial proximity and current popularity. This model brings together the geometric and dynamic mechanisms studied earlier and also shows why classical stabilization arguments must be adapted in growing networks. The last talks move toward current research on robustness and distances in spatial scale-free networks and on higher-order spatial models such as random connection hypergraphs.

Literature: The seminar will primarily use Remco van der Hofstad, Random Graphs and Complex Networks I and II; Mathew Penrose, Random Geometric Graphs; Massimo Franceschetti and Ronald Meester, Random Networks for Communication: From Statistical Physics to Information Systems; and Ronald Meester and Rahul Roy, Continuum Percolation. For stabilization and the more recent spatial network models, selected research articles will be used. Precise references are given with the individual topics below.

Times: Time and room will be announced separately. Regular attendance is required.

Prerequisites: Basic knowledge of probability theory is required. Familiarity with stochastic processes is helpful. No prior knowledge of stochastic geometry, point processes, percolation theory, or random graph theory is assumed.

Language: The preferred language of the seminar will be English.

List of presentations

1. Erdős-Rényi random graphs and branching-process approximation:
   Speaker: ______________________________
   van der Hofstad I, Sections 3.1-3.3 and 4.1-4.2, with emphasis on component exploration and the phase transition.

2. Erdős-Rényi random graphs: the giant component and connectivity threshold:
   Speaker: ______________________________
   van der Hofstad I, Section 4.4 and Sections 5.3-5.4 (selected parts).

3. Rank-one inhomogeneous random graphs: generalized random graphs and Norros-Reittu:
   Speaker: ______________________________
   van der Hofstad I, Sections 6.1-6.5 and Section 6.8, with emphasis on vertex weights and degree distributions.

4. Preferential attachment: growth, degree sequences and power laws:
   Speaker: ______________________________
   van der Hofstad I, Sections 8.1-8.4 and selected parts of Sections 8.5-8.7.

5. Poisson point processes and random geometric graphs:
   Speaker: ______________________________
   Penrose, Chapter 1; Franceschetti-Meester, Chapter 1 (selected parts).

6. Stabilization and laws of large numbers for spatial graph statistics:
   Speaker: ______________________________
   Penrose-Yukich, Weak laws of large numbers in geometric probability, Ann. Appl. Probab. 13 (2003), 277-303; selected examples from Penrose.

7. Continuum percolation: the Gilbert graph, Boolean model and random connection model:
   Speaker: ______________________________
   Franceschetti-Meester, Chapter 2; Meester-Roy, selected parts of Chapters 1-3 and the random connection model chapter.

8. Connectivity and component structure in finite random geometric graphs:
   Speaker: ______________________________
   Penrose, Chapters 9-10 and 13; Franceschetti-Meester, Chapter 3 (selected parts).

9. Weight-dependent random connection models: geometry meets heavy-tailed weights:

   Speaker: ____________________________

   Gracar-Lüchtrath-Mörters, Percolation phase transition in weight-dependent random connection models, Adv. Appl. Probab. 53 (2021), selected parts.

10. Geometric inhomogeneous random graphs (GIRGs):

    Speaker: ____________________________

    van der Hofstad II, Section 9.5.3; Bringmann-Keusch-Lengler, Geometric inhomogeneous random graphs, Theor. Comput. Sci. 760 (2019), 35-54.

11. Hyperbolic random graphs:

    Speaker: ____________________________

    van der Hofstad II, Section 9.5.2; selected parts of the hyperbolic random graph literature, with emphasis on degree heterogeneity and clustering.

12. Spatial preferential attachment: power laws, clustering and stabilization ideas:

    Speaker: ____________________________

    van der Hofstad II, Section 9.5.4; Jacob-Mörters, Spatial preferential attachment networks: power laws and clustering coefficients, Ann. Appl. Probab. 25 (2015), 632-662.

13. Distances and robustness in spatial scale-free networks:

    Speaker: ____________________________

    Selected research articles on weight-dependent and age-dependent random connection models, including Gracar-Grauer-Lüchtrath-Mörters, The age-dependent random connection model.

14. Higher-order spatial networks: random connection hypergraphs:

    Speaker: ____________________________

    Brun-Hirsch-Juhász-Otto, Random connection hypergraphs, Stochastic Models 42 (2026), 345-387, selected parts.

General information on seminar talks: About one week before your seminar talk, please arrange a meeting with me. Before this meeting you should have read and worked through the assigned material, decided on the main mathematical message of the talk, and prepared a preliminary structure. The meeting is intended to resolve remaining questions and to decide which results and proofs should be presented in detail.

A seminar talk should not attempt to reproduce the source line by line. The aim is to explain the central definitions, phenomena and arguments in a way that makes their role in the seminar visible. Results and proofs should receive priority, together with examples, figures, simulations or simple special cases that improve intuition. If a proof contains a long technical calculation, it is often better to explain the decisive idea carefully and omit routine details from the presentation, although you should have worked through those details yourself.

You should prepare your own notes rather than presenting directly from the source. In particular, think in advance about the amount of material that can realistically be written on the blackboard or shown in a small number of slides. The talk should be practiced beforehand so that there is enough time for the main argument and for questions. A useful rule is that one substantial theorem explained well is preferable to several results presented too quickly.

Some more pointers regarding a good seminar talk: You should understand the material well enough to place it in the larger context of the seminar. In particular, for the spatial topics you should be able to explain which features come from geometry and how the model differs from the non-spatial benchmark models discussed at the beginning. You should be ready to answer questions about notation, assumptions and the main steps of the arguments, or to explain precisely why a question goes beyond the scope of the talk.

Small gaps or implicit steps in the source should be filled in when this is feasible. Exercises, worked examples and simple pictures are often useful for this purpose. If a point remains genuinely unclear after careful preparation, it is better to say so explicitly than to hide the issue; this gives us an opportunity to clarify it together during the seminar.

The talk should begin with a short overview of its mathematical objective, its relation to previous talks, and the route to the main result. All notation and concepts used in the presentation must be introduced. I strongly recommend preparing a short handout containing the central definitions and results, together with reminders of notation from earlier talks when helpful. The main results of the talk should still be stated precisely during the presentation.