

Seminar on Statistical Extreme Value Theory

winter term 2022/23

Dr. Tobias Kley and Prof. Dr. Axel Munk

Key information

Time:	24/10/2022 – 10/02/2023, on Mondays, 16.15-17.45
Format:	Room HS 3 (Mathematisches Institut)
Possible Modules:	B.Mat.3444: Seminar on mathematical statistics M.Mat.4844: Seminar on mathematical statistics B.Mat.3447: Seminar on statistical foundations of data science M.Mat.4847: Seminar on statistical foundations of data science
Instructors:	Dr. Tobias Kley and Prof. Dr. Axel Munk
Intended Audience:	Advanced Bachelor and beginning Master students
Language:	English

Prerequisites

Participants must have successfully attended:

- Measure & probability theory (B.Mat.1400).

Description

For many tasks of modern data analysis the stochastic modeling of very rare events, such as observing extremely low/high values or a value larger/smaller than any of the previously observed values, is important in many applications. For example, consider modeling the height of tidal waves in the planning phase of building a dam, modeling of future claim heights when developing policies how much money insurance companies have to set aside, or modeling the oldest age or largest body height of people from a specific population.

For all of these questions the statistical behaviour, in particular the distribution, of the maximum $M_n = \max\{X_1, \dots, X_n\}$ of a sample X_1, \dots, X_n is to be considered. Central Limit Theorems (CLTs) allow us to approximate the distribution of many statistics that are (related to) the sum $S_n = X_1 + \dots + X_n$ obtained from a random sample. More precisely, under assumptions, we then have that $P((S_n - \mu_n)/\sigma_n \leq x) \rightarrow \Phi(x)$, as $n \rightarrow \infty$, where $\mu_n = E(S_n)$ and $\sigma_n^2 = \text{Var}(S_n)$. One of the aspects that makes CLTs so useful is that the limit is always $\Phi(x)$, the c.d.f. of the standard normal distribution. However, the behaviour of extremes is substantially different. Extreme Value Theory provides asymptotic results of the form $P((M_n - a_n)/b_n \leq x) \rightarrow G(x)$ and it turns out that



A man standing next to large ocean waves at Porto Covo, Portugal.

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G is always of the form

$$G(x) = \exp\left(-\left[1 + \xi\left(\frac{x - \mu}{\sigma}\right)\right]\right),$$

a so-called generalized extreme value distribution with location, scale and shape parameters μ, σ, ξ .

In this seminar we cover the basic relevant mathematical concepts and main results of extreme value theory. Our focus will be on carefully understanding the basic principles rather than obtaining results in most generality. Topics include: extreme value distributions, Cauchy's functional equation, Fisher-Tippett-Gnedenko theorem, regular variation, max-domain of attraction, and statistics for extreme value distributions. Finally, we discuss some applications and statistical estimation techniques for generalized extreme value distributions.

This seminar complements the lecture Foundations of Statistical Data Science I (B.Mat.3147) by Prof. A. Munk.

Application

To provide participants with the material to be presented at an early stage, we ask you to preregister for this seminar. To this end, please email Tobias Kley (tobias.kley@uni-goettingen.de) and indicate your interest to give a seminar talk. Please include information about relevant courses you have taken in your email. Deadline for preregistration is 26 September 2022 (12pm/noon).

A preparatory virtual meeting, during which topics will be assigned to participating students, is scheduled for 6 October 2022 (2pm–3pm). Notably, the seminar is limited to 14 participants. Should preregistrations exceed 14, then participants will be chosen based on the information provided in their preregistration email.

Recommended literature

Main Reference

Topics for presentations will be assigned along the lines of

- Z. Kabluchko: Lecture Notes “Extremwerttheorie”. Available for download via https://www.uni-muenster.de/Stochastik/kabluchko/Skripte/Skript_Extremwerttheorie.pdf

References for further reading

- P. Embrechts, C. Klüppelberg, T. Mikosch. Modelling Extremal Events for Insurance and Finance.
- S. Resnick. Extreme Values, Regular Variation and Point Processes.
- L. de Haan, A. Ferreira. Extreme Value Theory: An Introduction.
- S. Resnick. Heavy-Tail Phenomena: Probabilistic and Statistical Modeling.
- M. Falk, J. Hüsler, R.-D. Reiss. Laws of Small Numbers: Extremes and Rare Events.
- J. Beirlant, Y. Goegebeur, J. Segers, J. Teugels. Statistics of Extremes: Theory and Applications.