Seminar on
Statistical Aspects of Optimal Transport

Prof. Dr. Axel Munk

winter term 2025/26

Key information

Time:	31 Oct 2025 – 13 Feb 2026, on Fridays, 10.15-12.00
Format:	Room 5.101 (IMS)
Possible Modules:	B.Mat.3441: Seminar im Zyklus "Angewandte und Mathematische Stochastik"
	B.Mat.3444: Seminar im Zyklus "Mathematische Statistik"
	B.Mat.3445: Seminar im Zyklus "Statistische Modellierung und Inferenz"
	B.Mat.3447: Seminar im Zyklus "Statistische Grundlagen der Data Science"
	M.Mat.4841: Seminar on applied and mathematical stochastics
	M.Mat.4844: Seminar on mathematical statistics
	M.Mat.4845: Seminar on statistical modelling and inference
	M.Mat.4847: Seminar on statistical foundations of data science
Instructors:	Prof. Dr. Axel Munk and Jan Victor Otte
Intended Audience:	**Advanced Bachelor and beginning to intermediate Master students**
Language:	English

Prerequisites

Participants must have successfully attended

- Measure & probability theory (B.Mat.1400) (or have an equivalent qualification).

Intermediate knowledge of Statistics (e.g. Statistical Foundations of Data Science I) and/or of stochastics (e.g. B.Mat.2410) is recommended. Some understanding of optimization is helpful for some parts of the seminar but not necessary.

Description

Optimal transport (OT) can be thought of as the problem of transporting goods to some target at a minimum possible cost. Originally, it has been stated by G. Monge in the beginning of the 19th century in a variational formulation for mass preserving transport maps. Since then it has experienced a long and deep development in mathematics and related areas, such as physics and economic theory, reflected in L. Kantorovich's and T.C. Koopmans' Nobel prize in 1975. It has shaped and influenced many branches of mathematics including optimization, probability theory, and PDE analysis. Among others, two fields medals (C. Villani and A. Figalli) have been devoted to this area.

In its modern probabilistic formulation OT is stated as the problem to find a coupling (or a Markov kernel) between two probability measures with given marginals μ and ν such that the expected cost to transform μ into ν is minimized. This smallest expected cost to achieve the transport defines a distance on the set of distributions on an underlying (Polish) space. This distance takes into account the metric properties of this space to some extent. This yields the so called Wasserstein space and provides the foundation for a remarkably rich theory.

Alternatively, the optimal transport problem can be described using concepts from graph theory and optimization. More precisely, if the ground space is finite or countable, then each transport map between two sets of locations can be viewed as a matching in a weighted bipartite graph. Then, the optimal transport solution minimizes the sum of the edge weights, resulting in a specific linear program.

This paves the way for various new algorithms and due to computational progress OT has entered various fields of modern applications, ranging from cell biology to machine learning (e.g. (Tameling et al., 2021), (Peyré and Cuturi, 2019). Another corner stone for such applications is the recent advancement of the statistical understanding of OT (e.g. Panaretos and Zemel, 2020), i.e. when data are randomly sampled. This establishes optimal transport based data analysis as a valuable tool for many tasks in modern data science.

In this seminar we will first discuss some fundamentals of optimal transport in metric spaces, including its properties as a metric on the space of probability measures and some of its elementary geometric properties. For example, the so called transport plan (aka coupling) can be used to compute a geodesic in the Wasserstein space. We will discuss various examples and learn some fundamental algorithms to compute OT. In the second part we investigate the statistical behavior of optimal transport when data are randomly sampled. To this end, we learn how to analyze its convergence behavior when sample size grows and how this depends on the complexity of the underlying space. For example, if the space is euclidean, it will depend on the dimension in an explicit way. We will learn how much optimal transport estimation suffers from the "curse of dimensionality": Its worse case risk scales inversely proportional to the dimension. However, we will also show that this worst case scenario often can be overcome by optimal transport as it is able to adapt to the "inner Wasserstein dimension" of data, which can be much smaller than the ambient space. To this end we prove universal lower risk (information) bounds for empirical optimal transport, and show that simple estimation methods based on the empirical measure can achieve these bounds.

This insight into the statistical precision of optimal transport estimates can be used to control the behavior of randomized algorithms for its computation.

Application

To provide participants with the material to be presented at an early stage, we ask you to preregister for this seminar. To this end, please email Jan Otte (janvictor.otte@stud.uni-goettingen.de) and indicate your interest to give a seminar talk. Please include information about relevant courses you have completed in your email. The deadline for preregistration is **September 28, 2025**.

A preparatory virtual meeting, during which topics will be assigned to participating students, is scheduled for **October 2nd, 2025 (10:15-12.00)**. Notably, the seminar is limited to 13 participants. Should preregistrations exceed 13, then participants will be chosen based on the information provided in their preregistration email.

Recommended literature

Main Reference

Topics for presentations will be assigned along the lines of some chapters in

- Brezis, H. (2018), *Remarks on the MongeâKantorovich problem in the discrete setting*, C. R. Acad. Sci. Paris, Ser. I 356 (2), 207-213

- Santambrogio, F. (2015). *Optimal Transport for Applied Mathematicians: Calculus of Variations, PDEs, and Modeling*, Springer.
- Panaretos, V. M., and Zemel, Y. (2020). An Invitation to Statistics in Wasserstein Space. Springer. Available online: `https://link.springer.com/book/10.1007/978-3-030-38438-8`.
- Chewi, S., Niles-Weed, J., Rigollet, P., 2024, Statistical Optimal Transport, St Flour Lecutes, XLIX, https://arxiv.org/abs/2407.18163

References for further reading

- Bertsekas, D. P. (1991). *Linear Network Optimization: Algorithms and Codes*, MIT Press. Chapters 1–4 are vailable online: `http://web.mit.edu/dimitrib/www/net.html`.
- Bertsimas, D. and Tsitsiklis, J. N. (1997). *Introduction to Linear Optimization*, Athena Scientfic.
- Herrmann, M. (2020). *Graphentheorie.* Lecture Notes. Available online: `https://www.tu-braunschweig.de/ipde/personal/herrmann/skripte` (English translation will be made available on StudIP.)
- Peyré, G., and Cuturi, M. (2019). Computational Optimal Transport. Foundations and Trends in Machine Learning. Available online: `https://optimaltransport.github.io/`.
- Hundrieser, S., Staudt, T. Munk, A. (2024). *Empirical optimal transport between different measures adapts to lower complexity*, Annales de l'Institut Henri Poincare (B) Probabilites et statistiques 60, 824-846.
- Rachev, S. T., and Rüschendorf, L. (1998). *Mass Transportation Problems: Volume I: Theory.* Springer Science & Business Media.
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- Tameling, C., Stoldt, S., Stepahn, T., Naas, J., Jakobs, S., Munk, A. (2021). Colocalization for super-resolution microscopy via optimal transport. Nature Computational Science, 1, 199–211.
- Tarjan, R. E. (1983). *Data Structures and Network Algorithms*, SIAM. Available online: `https://epubs.siam.org/doi/book/10.1137/1.9781611970265`.
- Sommerfeld, M., Schrieber, J., Zemel, Y., Munk, A. (2019), *Optimal transport: Fast probabilistic approximation with exact solvers*, Journ. Mach. Learn. Res. 20(105), 1-23.
- Villani, C. (2009). *Optimal Transport: Old and New.* Springer, Berlin.