The Loewner Differential Equation and Stochastic Processes

The Loewner differential equation is a very simple ordinary differential equation that takes as an input a real-valued function and produces as an output a 2-dimensional set. It has been introduced in 1923 by Charles Loewner as a tool to study conformal maps (=analytic bijections) of a disc and was one of the key players in the celebrated proof of the Bieberbach conjecture by de Branges in 1985. Recently it has received enormous attention by analysts, probabilists, and physicists due to its key role in the resolution of various conjectures in a different area: Oded Schramm realized that numerous random sets previously studied by physicists and probabilists can be analyzed by running the Loewner equation with certain random real-valued functions (one-dimensional Brownian motion). This new stochastic process (now called the "Schramm Loewner Evolution" SLE) has been identified as the "limit" of various critical lattice processes (random walks, percolation) and is a constant source of beautiful theorems and conjectures.