# Some theoretical and practical notes about inference on semiparametric capture-recapture models

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Capture-recapture experiments are commonly used to estimate the size of a closed population. Link (2003) has underlined identifiability problems when one wants to make inference with heterogeneous capture probabilities in a semiparametric framework. If subject-specific capture probabilities are random effects with no assumption on the mixing distribution, the conditional likelihood is not identifiable. Link (2003) invokes the equivalence of conditional and complete likelihood (Sanathanan, 1972) to conclude that semiparametric inference is not possible in recapture studies. We show (i) that a regularity condition of Sanathanan (1972) is not met, and hence such equivalence does not hold; (ii) that the complete likelihood is indeed identifiable. Additionally, we prove the MLE is convergent but, surprisingly enough, not consistent. We characterize the limiting value of the MLE as a "sharpest estimable lower bound" and prove that the MLE can never over estimate the true population size. This is in parallel with the results of Holzmann et al. (2006) for the maximum of the conditional likelihood, and it establishes a new equivalence result. In practice, the complete likelihood parameter space includes all possible mixing distributions with support on [0,1], making inference cumbersome. We use the theory of canonical moments and a logistic transform to obtain a finite dimensional and unconstrained parameter space. We then use a Bayesian framework to fit the most general model in which heterogeneous detection probabilities are allowed also to depend on trapping occasion and behavioral reactions to first capture. We derive the Jeffrey's prior and illustrate the Bayesian and classical approaches with real examples and simulations.