Prof. Hoffmann: “The generic problem of estimating the smoothness of a blurred signal”

Abstract : We consider the problem of estimating the smoothness of an unknown signal, when we observe (for instance) a sampling of the signal corrupted by (for instance) white noise. Since any consistent notion of smoothness -global or local- is essentially asymptotic, the problem as stated is a little bit meaningless without further assumptions. Indeed, any de-noising method will tend to kill the relevant information about the smoothness, mostly contained in the high frequency part of the signal.

In a first part, we will review a body of work inspired by Cai and Low about constructing adaptive confidence intervals, a problem which is almost equivalent to estimating the smoothness of the signal through the duality between confidence intervals and hypothesis testing.

In a second part, we will specialize in specific class of signals, that "saturate" their smoothness in some sense, and that enable to overcome the limitations of the Cai and Low theory. This kind of saturation may be seen as putting a special nonparametric prior on the unknown function. If the signal is a reasonable transformation of a fractional Brownian motion, we will exhibit (min-max) optimal reconstruction of the smoothness.

In a last part, depending on the remaining time and on the audience, we will develop the previous approach to multifractal signals (as opposed to fractional Browian motion which is monofractal) in the toy model of multiplicative cascades of Mandelbrot, Kahane and Peyriere, observed in noise. The underlying smoothness is then charactrized by the so-called structure function, which translates into the signal belonging to the intersection of Besov balls by means of the works of Jaffard on the Firsh-Parisi conjecture.

The novel results presented here are joints with Arnaud Gloter.